THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MMATH5220 Complex Analysis and Its Applications 2014-2015
Assignment 5

- Due date: 15 Apr, 2015
- Remember to write down your name and student number

1. Let $z_1, z_2, \ldots, z_n$ be distinct complex numbers. Let C be a circle around z_1 such that C and its interior do not contain z_j for $j > 1$. Let
$$f(z) = (z - z_1)(z - z_2) \cdots (z - z_n).$$
Find
$$\int_C \frac{1}{f(z)}\, dz.$$

2. Use residues to show that
   (a) $\displaystyle\int_0^\infty \frac{x^2 dx}{(x^2+9)(x^2+4)^2} = \frac{\pi}{200}$
   (b) $\displaystyle\text{P.V.}\int_{-\infty}^\infty \frac{\sin x}{x^2+4+5}\, dx = -\frac{\pi}{e}\sin 2$
   (c) $\displaystyle\int_0^\pi \frac{d\theta}{(a+\cos\theta)^2} = \frac{a\pi}{(\sqrt{a^2-1})^3}$, where $a > 1$
   (d) $\displaystyle\int_0^\infty \frac{x^a}{(x^2+1)^2}\, dx = \frac{(1-a)\pi}{4\cos(a\pi/2)}$, where $-1 < a < 3$

3. Suppose that a function f is analytic inside and on a positively oriented simple closed contour C and that it has no zeros on C. Show that if f has n zeros z_k $(k = 1, 2, \ldots, n)$ inside C, where each z_k is of multiplicity m_k, then
$$\int_C \frac{zf'(z)}{f(z)}\, dz = 2\pi i \sum_{k=1}^n m_k z_k.$$

4. Use Rouché's theorem to show that $z^5 + 3z^3 + 7$ has all it's zeros in the disk $|z| < 2$.